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Structured Ramp Secret Sharing Schemata

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Abstract

Secret sharing schemata using computational techniques of Linear Algebra are proposed and analyzed. Specifically, (s, t, n) - threshold *Ramp Secret sharing* schemata are presented, where s and t are positive integers, while n denotes the number of *participants* of a secret sharing scheme. Any set of h participants, $0 < h < s$, are not able to retrieve any information about the secret, while any set of h' participants, $s \leq h' < t$, are able to retrieve only a part of the secret and any set of h'' participants, $t \leq h'' \leq n$, retrieve the whole secret. The person that manipulates the secret sharing procedure, named *dealer*, constructs a polynomial $p(x)$ of degree k , in which the secret is properly hidden. Next, the dealer creates a set of r *structured levels* L_i , $i = 1, 2, \dots, r$ and shares a polynomial to each participant at every level. Several specified by the dealer participants from levels L_i , L_j , $i, j \in \{1, 2, \dots, r\}$, $i \neq j$, must cooperate revealing their shares (represented by polynomials) and compute the *Greatest Common Divisor* of polynomials to retrieve the secret. To compute the greatest common divisor of polynomials, the participants can use direct methods for matrices factorization, for instance the well-known and widely used LU or QR factorization procedures. Our approaches are studied in respect of error analysis and complexity. Furthermore, indicative numerical examples are presented to facilitate the proposed schemata.

References

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