Archs oral Biol. Vol. 32, No. 8, pp. 579–586, 1987
Printed in Great Britain.

0003-9969/87 $3.00 + 0.00

A MATHEMATICAL DESCRIPTION OF THE COMMINUTION OF FOOD DURING MASTICATION IN MAN

A. van der Bilt, L. W. Olthoff, H. W. van der Glas, K. van der Weelen and F. Bosman

Department of Oral Pathophysiology, University of Utrecht, P.O. Box 80.080, 3508 TB Utrecht, The Netherlands

Summary—Chewing performance was quantified by determining the particle-size distribution of comminuted food as a function of the number of chewing strokes. The rate of food breakdown was taken to be the result of a combined selection and breakage process; this was quantified in a mathematical model. A linear operation on the particle-size distribution described the changes in this distribution that resulted from an additional chewing stroke. Detailed information was obtained from eight subjects on the selection and breakdown of food particles of different sizes. There were considerable inter-individual differences in the selection chances for small particles. The mathematical method facilitates study of the influence of dental morphology and muscle-related factors on the comminution of food particles.

INTRODUCTION

The comminution of solid food is of interest in the study of the masticatory system and of the chewing performance of natural and artificial human dentitions (see Dahlberg, 1942; Manley and Braley, 1950; Edlund and Lamm, 1980; Lucas and Luke, 1983a; Olthoff *et al.*, 1984). This performance has been quantified by determining the particle size of comminuted food by test sieving, and this has proved an appropriate method. The distribution of particle size in comminuted food is adequately described by a Rosin–Rammler distribution function which is characterized by two variables only (Olthoff *et al.*, 1984). By this means, the outcome of chewing can be quantified. How the comminution of different sized particles when mixed in the mouth takes place remains to be described.

To analyse the communition of materials in industrial processes, Epstein (1947) introduced the concept that the rate of breakdown of a material is the result of two processes, selection and breakage. This has been applied to the analysis of industrial comminution processes (Broadbent and Callcott, 1956) and of food comminution (Lucas and Luke, 1983a; van der Glas *et al.*, 1985). During each chewing cycle, a food particle has the chance of being placed between the teeth (selection) and then being fractured into fragments of variable number and size (breakage). Particles are considered to be selected if they are comminuted or at least damaged by the teeth. Selection for breakage may depend upon factors like jaw, tongue and cheek movements, the total occlusal area of the molar teeth, tooth shape, particle size, and the total amount of food in the mouth. Breakage may depend upon tooth shape, maxillomandibular relationships and the intensity and coordination of the jaw-muscle activity which is generating the bite-force. The texture of the food may influence both processes (Olthoff *et al.*, 1986).

Lucas and Luke (1983a) determined the selection chances for single-size classes of carrot particles by either staining particles of one size class or by giving these particles a distinct form (cylinders). In the experiments of van der Glas *et al.* (1985) particles of silicon rubber of seven different size classes were labelled by colour and by form (cubes). Selection and breakage could be determined simultaneously for particles of these size classes.

A mathematical model of the selection and breakage of particles in industrial comminution processes was introduced by Broadbent and Callcott (1956) and Berenbaum (1961). This method, using matrix algebra, enables description of the changes in particle-size distribution during comminution. The matrix also describes how particles of different sizes are comminuted. To calculate this matrix, the particle-size distributions in various phases of chewing must be known; the breakage of particles must then be determined in a calibration experiment. The results from the matrix model can be verified experimentally by the double-labelling method of van der Glas *et al.* (1985).

Our aim was to quantify food communition in terms of the selection and breakage of food particles using the matrix model.

MATERIALS AND METHODS

Eight males, age range from 20 to 39 years, volunteered and gave informed consent. All had complete natural dentitions and none showed any signs of the myofacial pain-dysfunction syndrome. Eight cubes of green Optosil (Bayer; stock 1980), a silicon rubber commonly used as a dental impression material, with an edge length of 8 mm, were used as a test food. After chewing a fixed number of strokes, N, the particle-size distribution of the chewed food was determined by sieving. In different experiments, the number of chewing strokes ranged between 10 and 120 cycles. A detailed description of the experimental procedure and the analysis of particle-size distribu-

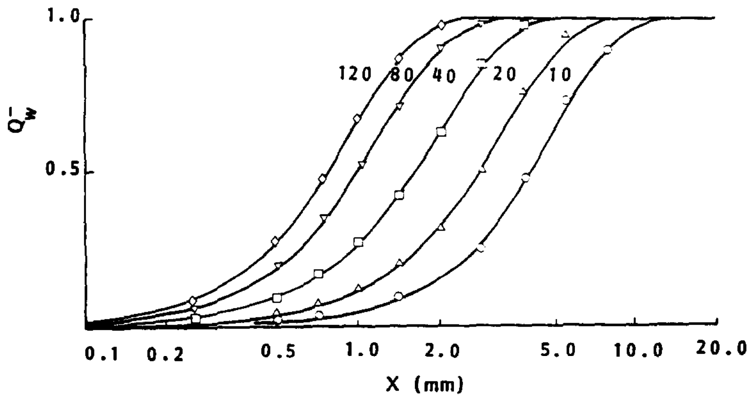

Fig. 1. Cumulative weight-fraction undersize Q_w^- as a function of the logarithm of the sieve aperture x (mm) for individual 7 after various numbers of chewing strokes. Drawn lines are best fits through the data points according to equation (1).

tions has been published (Olthoff *et al.*, 1984); only an outline will be given here.

Particle-size distribution

The weight of the particles on each sieve can be represented in a plot which shows the cumulative weight-percentage undersize as a function of the sieve aperture. The example gives data points plotted for subject no. 7 after 10, 20, 40, 80 and 120 chewing strokes (Fig. 1). The cumulative particle-size distribution is adequately described by a distribution function (Rosin and Rammler, 1933; Olthoff *et al.*, 1984):

$$Q_w^-(x) = 1 - 2^{-(x/x_{50})^b} \qquad (1)$$

where $Q_w^-(x)$ is the weight fraction of particles with a size smaller than x; the median x_{50} is the aperture of a theoretical sieve through which 50 per cent of the weight can pass, and b indicates the extent to which the particles are equally sized. Increasing values of b correspond to curves with steeper slopes and thus to distributions of particle sizes that are less broad. The variables x_{50} and b were determined by curve-fitting the data on equation (1) by a least-squares method (Bevington, 1969). The particle-size distribution cannot be fitted successfully at the start of chewing because hardly any particles have been broken and all of them are about the same size, which gives a step-like size distribution. However, good fits can be obtained after chewing at least ten times (Olthoff *et al.*, 1984). The scatter of the data points around the fitting curve was of the same order as the experimental accuracy of these points (1 per cent). The accuracy of the determination of x_{50} and b was of the order of 2 per cent for all fits. The dependence of variable b on the number of chewing strokes, N, was rather weak. The median particle size, x_{50}, decreases as a function of the number of chewing strokes according to the relation:

$$x_{50}(N) = c \cdot N^{-d} \qquad (2)$$

The variable c defines the notional median particle size after one chewing stroke. This value is only theoretical because relevant data for x_{50} can only be obtained after at least ten chewing cycles. The decrease of the median particle size per chewing stroke is given by variable d. A large value of d corresponds to a large decrease in particle size per chewing stroke, and therefore there will be relatively small median particle sizes at the end of the chewing process. To quantify chewing efficiency so that both variables c and d contribute, we define it as the number of chewing strokes needed to halve the initial median particle size of the 8 mm cubes, denoted as N (4 mm). This value can be calculated directly from equation (2) using the values of c and d. Some examples of data points fitted with relation (2) are given in Fig. 2.

Matrix model

The outcome of the comminution process is adequately described by equations (1) and (2), but no information is given about how comminution takes place, or how well the food particles of various size-classes are selected and fragmented during each chewing cycle. Such information can be obtained by studying how the distribution of particle sizes is changing during one arbitrary chewing cycle.

For this, the particle size distribution after N and $N + 1$ chewing cycles must be considered, so that the transition between the two distributions can be studied. The value of the median, x_{50}, can be calculated from equation (2) for any number of chewing movements, N, within the range 10 to 120, once the variables c and d are determined. The variable b can be obtained by interpolation between experimentally-determined values. Thus, from equation (1), any particle-size distribution can be calculated within this range of chewing cycles. Figure 3 shows schematically the changes in particle-size distribution between N and $N + 1$ chewing cycles and how this may be represented by a matrix of numbers. This comminution matrix **A**, describes the effect of one chewing stroke on the original, or feed, distribution of particles, **f**, giving the final, or product, distribution, **p**.

Theoretically, the comminution matrix cannot be

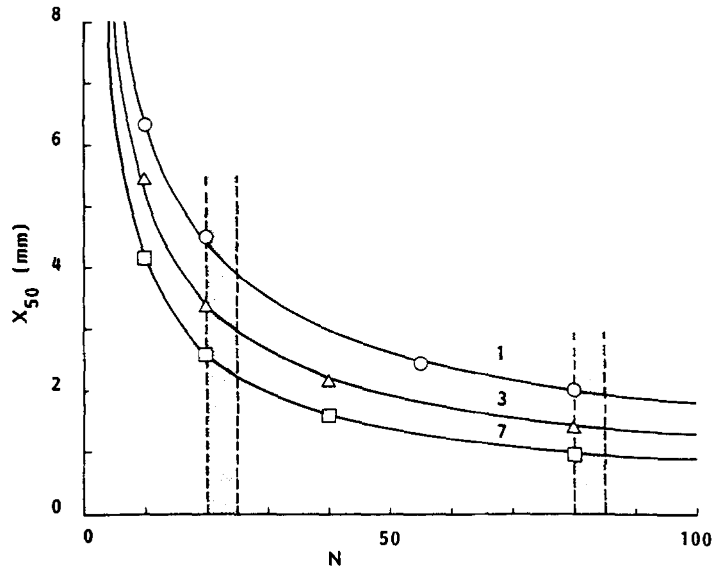

Fig. 2. Calculated values of the median x_{50} (mm) plotted versus the number of chewing strokes N for subjects 1, 3 and 7. Drawn lines are best fits through the data points according to equation (2).

calculated directly from the two known distributions **f** and **p** of a vector length $\geqslant 3$ because there are too many unknown variables $a_{i,j}$. However calculation becomes possible if assumptions are made about the dependence of selection and breakage on particle size. The matrix can be split into 2 parts, a matrix **S** describing the selection process, and a matrix **B** describing the breakage of the selected particles. Two assumptions are made:

(1) The probability of a particle being selected for breakage varies according to a power function of its size, x:

$$S(x) = v \cdot x^w \tag{4}$$

The exponent w determines how strongly the chance of selection depends upon particle size. If $w = 2$, the probability of selection of a particle increases four times for each doubling of the particle size. The variable v represents the chance of selection of a particle of unit size, x (equal in our experiments to 1 mm). In an experiment with three sieves only, if x_1, x_2 and x_3 are the average sizes of particles on successive sieves, the selection matrix will be

$$\mathbf{S} = v \begin{pmatrix} x_1^w & 0 & 0 \\ 0 & x_2^w & 0 \\ 0 & 0 & w_3^w \end{pmatrix} \tag{5}$$

Multiplying the feed distribution, **f**, by this matrix, converts the elements f_1. f_2, f_3 to $vx_1^w f_1$, $vx_2^w f_2$, and $vx_3^w f_3$. These represent the part of the feed distribution selected for breakage.

(2) The degree of fragmentation of a selected particle is independent of its size before breakage. Therefore each particle will, on average, be fragmented into the same number of fragments in the same ratio of sizes regardless of the initial size of the particle. The weight fractions of the fragments of a particle of initial size x_0 can be adequately described by a cumulative distribution function (Austin, 1971; Gaudin and Meloy, 1962):

$$B(x) = 1 - (1 + rx/x_0)(1 - x/x_0)^r \tag{6}$$

where $B(x)$ is the weight fraction of selected particles of initial size x_0 which break into particles smaller than size x, and r is related to the degree of fragmentation. The value of r increases as fragmentation increases, but is assumed to be independent of initial particle size. The breakage function $B(x)$ was determined experimentally for each subject by chewing once on an 8 mm cube of Optosil. The experiment was repeated 100 times to obtain a reliable size distribution for the fragmented particles and therefore a good estimate of the fragmentation variable r. The weight fractions of fragments on successive sieves could then be calculated for any initial particle size using equation 6 with this r value. A breakage matrix **B** for an experiment using 3 sieves can be constructed

$$\mathbf{B} = \begin{pmatrix} b_{1,1} & b_{1,2} = 0 & b_{1,3} = 0 \\ b_{2,1} & b_{2,2} & b_{2,3} = 0 \\ b_{3,1} & b_{3,2} & b_{3,3} = 1 \end{pmatrix} \tag{7}$$

in which $b_{1,1}$, $b_{2,1}$ and $b_{3,1}$ are the fractions of fragments originating from the particles on the top sieve, calculated from equation 6, $b_{2,2}$ and $b_{3,2}$ are the corresponding fractions from the second sieve, and $b_{3,3}$ is the fraction which remains on the third sieve. Fractions $b_{1,1}$ and $b_{2,1}$ are particles which have been damaged rather than broken so that they remain on their original sieve.

The comminution matrix **A**, can be expressed in terms of the selection and breakage matrices, **S** and **B**:

$$\mathbf{A} = [\mathbf{BS} + (\mathbf{I} - \mathbf{S})] \tag{8}$$

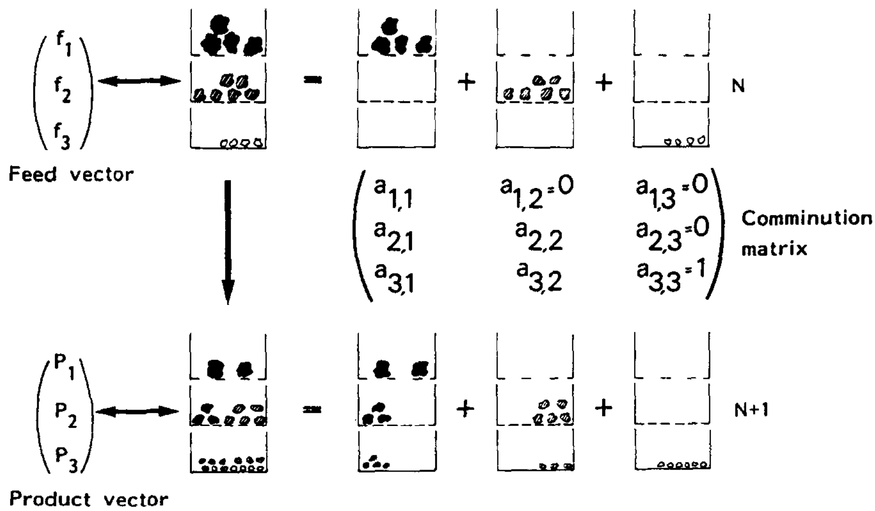

Fig. 3. Schematic view of the transition in particle-size distribution between N and $N+1$ chewing cycles. The distribution of particle sizes after N chewing cycles is depicted in the upper left of the figure as particles on three sieves of consecutively smaller mesh size. The effect of an additional chewing movement on particles of the three size classes is shown in the lower left of the figure. For clarity only three size classes are shown, although ten were used in the experiment. The three weight fractions of this distribution are shown in the column on the left as f_1, f_2 and f_3. Thus f_1 is the weight fraction on the upper sieve after N chewing cycles. The column of numbers (a vector in matrix algebra) represents the size distribution after N strokes; it will be termed here the feed distribution **f**. The column of numbers, p_1, p_2, p_3, in the lower part of the diagram, representing the distribution of sizes after $N+1$ strokes, will be termed the product distribution **p**.

On the right of the diagram the effects of the extra chewing stroke on the contents of each sieve are shown separately. The three fractions produced from the top sieve (the largest size class) as a result of the chewing stroke are designated $a_{1,1}$, $a_{2,1}$ and $a_{3,1}$. If the total weight of the particles remains constant, then $a_{1,1}+a_{2,1}+a_{3,1}=1$. The second largest size class is comminuted to three fractions $a_{1,2}$, $a_{2,2}$ and $a_{3,2}$, but the first of these is equal to 0 because no particles can be larger than those before the additional chewing stroke. Similarly, in the third group, both $a_{1,3}$ and $a_{2,3}$ must equal 0, and so $a_{3,3}=1$ (that is, all the smallest class of particles remain in the smallest class). The results of the chewing cycle are shown in the diagram as the comminution matrix **A**.

The product distribution p_1, p_2 and p_3 can now be predicted from the comminution matrix and the original feed distribution:

$$\begin{aligned} a_{1,1}f_1+a_{1,2}f_2+a_{1,3}f_3&=p_1\\ a_{2,1}f_1+a_{2,2}f_2+a_{2,3}f_3&=p_2\\ a_{3,1}f_1+a_{3,2}f_2+a_{3,3}f_3&=p_3 \end{aligned} \tag{3a}$$

where $a_{i,j}$ is the fraction of food particles that originates from sieve j and f_i and p_i are the weight fractions of the feed and product distributions on sieve i. The matrix in 3a can be expressed as

$$\begin{pmatrix} a_{1,1} & a_{1,2}=0 & a_{1,3}=0\\ a_{2,1} & a_{2,2} & a_{2,3}=0\\ a_{3,1} & a_{3,2} & a_{3,3}=1 \end{pmatrix}\begin{pmatrix} f_1\\ f_2\\ f_3\end{pmatrix}=\begin{pmatrix} p_1\\ p_2\\ p_3\end{pmatrix} \tag{3b}$$

or, more simply, as

$$\mathbf{Af}=\mathbf{p} \tag{3c}$$

where **A** is the comminution matrix with elements $a_{i,j}$, **f** is the feed distribution, and **p** is the product distribution.

The term **BS** represents the breakage of the selected particles and **I** − **S** represents the portion of feed not selected for breakage. Matrix **I** is the unit matrix; multiplication of a distribution by matrix **I** leaves the distribution unchanged.

The elements of matrix **B** can be calculated from the experimentally-determined fragmentation value r. Matrix **S** can be expressed using the two unknown variables v and w. Matrix **A** can therefore be constructed from equation 8 and will contain these two variables. From a known feed distribution, **f**, a theoretical product distribution, $\mathbf{p}_t$, was calculated by assigning values to the selection variables v and w. These values were then varied to minimize the differences between the theoretical and the experimentally-determined product distribution, $\mathbf{p}_e$.

The optimal values for v and w were determined by a least squares method using a HP-1000 mini-computer system.

The matrix **A** thus determined is related to one chewing cycle only. However, it is also possible to describe the effect of N chewing strokes on the mixture of particles by multiplying the feed distribution **f** by the matrix **A**, N times to give the product distribution **p**:

$$\mathbf{A}^N\mathbf{f} = \mathbf{p}.$$

Thus the effect of several chewing strokes can be described. The matrix model can be used both for distributions generated using a Rosin–Rammler function, and for distributions determined experimentally.

RESULTS

Particle-size distributions

In Fig. 1, the solid lines correspond to the Rosin–Rammler curves, equation (1), which best fit the data points obtained after sieving. The curves shift towards smaller sieve sizes with the increasing number of chewing strokes, N. The slope of the curves depends only slightly on N, which results in an almost constant value for the variable b.

The median particle size, x_{50}, obtained from the Rosin–Rammler curves are plotted in Fig. 2 as a function of N. For clarity, the results for three subjects only (1, 3 and 7) are shown, individuals with poor, intermediate and good chewing efficiency. The lines are best fits for the data points according to equation (2). The values of the variables c and d in this equation, and the chewing efficiency parameter, N (4 mm), are listed in Table 1 for all subjects. Marked differences in the particle-size reduction were observed among the various subjects. For instance, subject 1 required nearly 24 chewing cycles to reduce the median particle size from 8 to 4 mm, whereas subject 6 had to chew only six times to do this.

Breakage

The breakage functions after one bite are depicted in Fig. 4; only the results for subjects 1, 3 and 7 are shown. The cumulative weight-fraction undersize, B, is plotted as a function of the relative particle size, x/x_0. The data points are mean values from 100 measurements. The lines are best fits through the data points according to equation (6). The values of the fragmentation variable, r, are given in Table 1 for all subjects; a larger value of variable r denotes more fragmentation; this is reflected in larger values for the undersize, B, at a given relative particle size x/x_0 (Fig. 4).

Selection

The unknown selection variables, v and w (equation 4), were determined by a least-squares method. Optimal values for v and w were calculated for an early chewing phase (between cycle 20 and 25) and for a later one (between cycle 80 and 85; shaded areas in Fig. 3); these results are given in Table 1. The chance that a particle would be selected for breakage during one chewing cycle, S, is plotted as a function of particle size, x, in Fig. 5. The differences between the early and the late phase were small, though larger selection values were observed in the late phase for subject no. 7. There were large interindividual differences in the ability to select particles for breakage.

Comminution matrix

This matrix, **A**, can be calculated for any chewing cycle within the experimental range ($10 \leqslant N \leqslant 120$) by using the interpolated values of the variables x_{50} and b together with the experimentally-determined breakage function. The differences between the theoretical and the experimental product distributions were less than 1 per cent for all subjects. This indicates that the product distribution can be adequately calculated with the matrix model. Examples of comminution matrices are given in Table 2. These describe the transition in particle-size distribution between 20 and 25 chewing cycles for subjects 1 and 7. Because these matrices describe the effect of five chewing cycles, the figures in Table 2 refer to the fifth power of the comminution matrix for one chewing cycle ($\mathbf{A}^5$). For clarity, only six size classes are shown instead of the ten used in the experiments. The six

Table 1. Values for various variables for all subjects

					20–25		80–85	
Subject	c (mm)	d	N (4 mm)	r	v	w	v	w
1	22.7	0.55	23.5	0.44	0.004	2.0	0.006	1.8
2	26.1	0.61	21.6	0.24	0.012	1.9	0.014	1.9
3	21.0	0.61	15.2	0.38	0.014	1.7	0.014	1.7
4	26.7	0.78	11.4	0.45	0.048	1.0	0.036	1.2
5	17.2	0.66	9.1	0.50	0.024	1.6	0.027	1.7
6	12.2	0.60	6.4	0.45	0.041	1.3	0.028	1.4
7	20.9	0.70	10.6	0.21	0.074	1.2	0.063	1.0
8	19.4	0.57	16.0	0.18	0.020	1.9	0.023	1.8

c and d: variables describing the dependence of the median particle size, x_{50}, on the number of chewing strokes N (equation 2).
N (4 mm): number of chewing strokes needed to halve the initial median particle size.
r: variable related to the degree of fragmentation (equation 6).
v and w: variables describing the dependence of the selection chance on the particle size x (equation 4) for an early phase, cycle 20–25, and a late phase, cycle 80–85, of the chewing process.

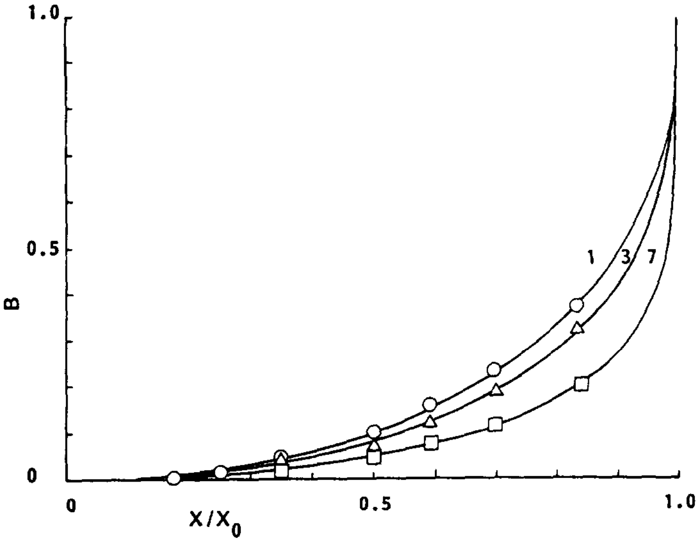

Fig. 4. Cumulative weight fraction undersize B as a function of the relative particle size x/x_0 for subjects 1, 3 and 7. Drawn lines are best fits through the data points according to equation (6).

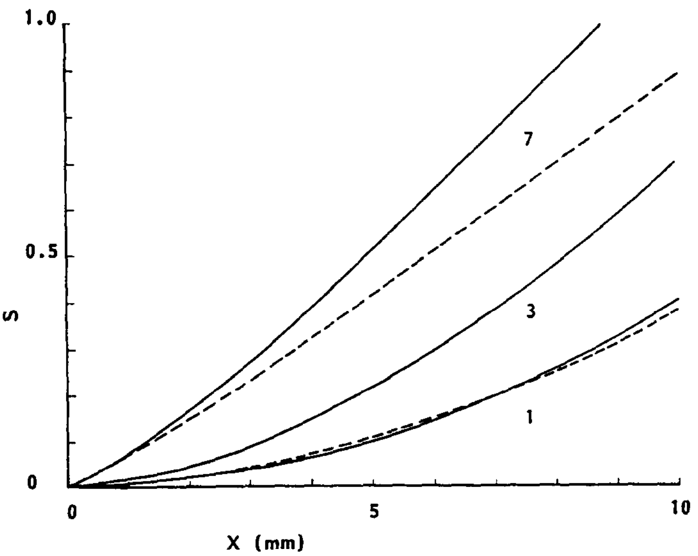

Fig. 5. Selection chance, S, for breakage per chewing cycle as a function of particle size x (mm) for subjects 1, 3 and 7 according to equation (4). The solid lines refer to an early phase, 20–25 chewing cycles, and the dashed lines to a late phase of the chewing process, 80–85 cycles. For subject 3 both lines coincide so the dashed line is omitted.

columns in the matrix show how the particles of six different size classes were fragmented during these five chewing cycles. For subject 1, 39 per cent of the particles of size class 8 mm were not fragmented, but 31 per cent of the 8 mm particles were comminuted into the size class of 5.6 mm. The percentage of particles either not affected, or only slightly damaged, during the five chewing strokes increased for the smaller sizes. Thus, 81 per cent of the 4 mm particles remained in their original size-class and 95 per cent of the 2 mm particles were not fragmented. The particles which were on the bottom of the sieve stack (size class 0.0) will all stay there, and the corresponding fraction will therefore be 1. The results for subject 7 show that the fraction of particles which remained in the initial size class was smaller for all sizes; therefore a larger fraction of the particles was comminuted.

DISCUSSION

Test food

The mathematical model provides quantitative information about the comminution of the particles of different sizes. To obtain reproducible results a standardized test food must be used. Optosil was chosen because both the form and consistency of particles can be more easily reproduced than with natural

Table 2. Comminution matrix A^5 corresponding to the transition in particle size distributions between 20 and 25 chewing cycles for subjects 1 and 7

	8.0	5.6	4.0	2.8	2.0	0.0
Subject 1:						
8.0	0.39	0	0	0	0	0
5.6	0.31	0.65	0	0	0	0
4.0	0.15	0.19	0.81	0	0	0
2.8	0.08	0.09	0.12	0.90	0	0
2.0	0.04	0.04	0.04	0.06	0.95	0
0	0.03	0.03	0.03	0.04	0.05	1
Subject 7:						
8.0	0.28	0	0	0	0	0
5.6	0.33	0.46	0	0	0	0
4.0	0.19	0.27	0.60	0	0	0
2.8	0.10	0.14	0.23	0.73	0	0
2.0	0.05	0.06	0.09	0.16	0.81	0
0	0.05	0.07	0.08	0.11	0.19	1

Columns refer to six different size classes and give information about the fragmentation into smaller particles due to five chewing cycles. The numbers in a column refer to the weight fractions retained on the various sieves.

substances like peanut and carrot. The reproducibility of the comminution results are much better for Optosil than for natural foods (Olthoff *et al.*, 1984, 1986). Furthermore, Optosil is not softened by the saliva during chewing as this silicon rubber is hydrophobic.

Matrix model

The comminution matrix was calculated for various pairs of size distribution with assumptions (see Materials and Methods) concerning the selection and breakage of particles. The validity of these assumptions was tested experimentally by van der Glas *et al.* (1985). In their study, the selection and breakage of Optosil particles were determined simultaneously for four subjects who went on to participate in our study (1, 4, 6 and 8). Particles of seven different size classes were labelled by colour and by form. Analysis of the chewed particles enabled determination of both selection and breakage so that a comminution matrix could be constructed.

Breakage

In the mathematical model, it is assumed that the breakage of particles is identical for all sizes. With this assumption, Broadbent and Callcott (1956) applied the model successfully to industrial comminution processes. Lucas and Luke (1983a) also assumed that particles of any size were equally broken and therefore they measured a breakage function for particles of the largest size class only. From their results for the selection chances and this breakage function, Lucas and Luke (1983b) could generate particle-size distributions from a computer program which modelled mastication. Labelling experiments by van der Glas *et al.* (1985), however, revealed that the breakage function depends on particle size, larger particles being better fragmented. To take account of the size-dependent breakage we suggest that experimental breakage functions should be determined for several particle sizes. Then, a breakage matrix can be constructed which would be more in accord with the results of the colour-labelling experiments.

Although the fragmentation variable r varies as a function of particle size, the double-labelling method gave r-values for particles of 8 mm which were in accord with our values from the four subjects. Substantial fragmentation of 8 mm particles was achieved by subject 5 ($r = 0.50$), whereas subject 8 had a poor performance ($r = 0.18$). Factors which influence breakage are post-canine tooth morphology, the chewing force and the direction of the mandibular movement in crushing the food. Teeth with more pronounced cusps may achieve better fragmentation; this assumption was verified in a study of the textural properties of Optosil (Olthoff *et al.*, 1986), in which particles were crushed in a machine with tooth-like plungers, and fragmentation improved when plungers with a smaller angle were used. Another factor influencing breakage is the force applied during chewing. The force needed for crushing one 8 mm particle is about 50 N (Olthoff *et al.*, 1986); chewing on several particles of that size will require a multiple of this force. However, only a fraction of the particles will be selected in each chewing cycle. Mean maximum bite-forces for people in the West are reported to be about 500 N (Carlsson, 1974). Thus, the forces needed to crush Optosil are well within the physiological range of our subjects. Also the bucco-lingual component of the jaw movement during food engagement may influence the degree of fragmentation of the food (Ardran and Kemp, 1960; Ahlgren, 1976).

Selection

Labelling experiments (Lucas and Luke, 1983a; van der Glas *et al.*, 1985) show that the chance to select a food particle for breakage indeed increases as a power function of the particle size, confirming the assumption made in our model (equation 4). In the experiments of Lucas and Luke, the exponent w varied between 1.62 and 1.85, which is in our range of w-values (Table 1). The results of van der Glas *et al.* (1985) were also in accord with our values of v and w for the same feed and product distributions.

The chances to select food varied between the subjects; moderate differences were observed for the largest particles (8 mm). The selection chance per chewing cycle varied between 26 per cent for subject 1 and 90 per cent for subject 7 (Fig. 5; Table 1 and equation 4). However, large differences in S occurred for small particles, the chance to select a particle of 1 mm varied between 0.4 per cent for subject 1 and 7.4 per cent for subject 7. These large differences may be explained by factors related to the morphology of the teeth and to the transport of food particles about the mouth. In subject 1, the inter-occlusal relationship was defined as a wide-centric occlusion without stability; this type of occlusion, caused by rounded and flattened occlusal restorations in the premolar–molar region, may hamper the selection of small particles for breakage. In subject 7, a close intercuspal jaw relationship in centric occlusion facilitated the selection of small particles. Fragments may be further reduced into smaller particles when they are retained between the teeth by occlusal cusp height and fossa depth. The selection chances in the early and late phases of chewing are more or less the same, as can be seen from the variables v and w in Table 1. Apparently the selection chance is not influenced by different particle-size distributions as long as the total amount of food is constant.

A factor in the reduction of the selection chance for smaller particles is the increase in the total projected area of small particles, whereas the tooth area available for breakage is constant. Suppose a particle is fragmented into pieces that are a factor n smaller; the number of fragments will then be n^3, and the area needed for one fragment to be broken between the occlusal surfaces will be $1/n^2$ of the original particle. The total occlusal area needed for all fragments will therefore be n times the initial area. When this area becomes larger than the total occlusal surface available for breakage, the selection chance will decrease and a linear dependence of S as a function of particle size can be expected. Indeed, values for the exponent w of equation (4) close to 1 were obtained for subjects 4, 6 and 7; larger values for the other subjects indicate a greater decrease in selection chance for smaller particles, and thus, the actual number of particles between the teeth is smaller than can be theoretically expected. This may be caused by a different transport about the mouth for small particles, or a smaller tooth area involved in breakage of those particles.

Masticatory efficiency

This showed marked interindividual differences as shown by the number of chewing cycles needed to halve the initial particle size, N (4 mm) (Table 1). These must originate in either a different ability to select food particles for breakage, a different breakage function or in a combination of these factors. Small values of N (4 mm) were found for subjects 4, 5, 6 and 7, indicating a good chewing performance; however, subject 7 has a small r value (0.21) indicating poor breakage. Apparently poor breakage capability can be compensated for by good selection. Indeed, large selection chances were found for subject 7 over the whole range of particle size. The poorest chewing ability was by subject 1: this was due to low selection chances, as the fragmentation variable r was high (0.44).

Comminution matrix

Either experimentally-determined distributions or the corresponding Rosin–Rammler distributions can be used. As the size distributions are adequately described by the Rosin–Rammler equation, both methods lead to similar results. However, the Rosin–Rammler distribution has the advantage that it offers the possibility of generating size distributions at any number of chewing strokes in the range 10 to 120 by interpolation of the variables x_{50} and b. There was good agreement between the calculated matrices and the corresponding matrices determined from the experimental data of van der Glas *et al.* (1985). The differences in corresponding elements, $a_{i,j}$, of both matrices were of the same order as the experimental scatter (2 per cent).

The matrix model enables quantitative description of the comminution process. It can be used as a tool to study the influence on comminution of dental and muscle-related factors. The effect of dental restorations on the chewing efficiency can also be studied quantitatively. Furthermore, it facilitates the determination of optimal cusp-fossa anatomy for partial and complete dentures by objective criteria.

Acknowledgements—We wish to thank Professor Dr B. Scarlet and Dr W. H. A. Steen for their valuable discussions and for a critical reading of the manuscript.

REFERENCES

Ahlgren J. (1976) Masticatory movements in man. In: *Mastication* (Edited by Anderson D. J. and Matthews B.) pp. 119–127. Wright, Bristol.

Ardran G. M. and Kemp F. H. (1960) Biting and mastication. A cineradiographic study. *Dent. Pract. Dent. Rec.* **11**, 23–36.

Austin L. G. (1971) A review; introduction to the mathematical description of grinding as a rate process. *Powder Technol.* **5**, 1–17.

Berenbaum R. (1961) Analysis of the shatter breakage of coal using a matrix model. *J. Inst. Fuel* **34**, 367–374.

Bevington P. R. (1969) *Data Reduction and Error Analysis for the Physical Sciences*. McGraw-Hill, New York.

Broadbent S. R. and Callcott T. G. (1956) I. A new analysis of coal breakage processes. II. A matrix representation of breakage. *J. Inst. Fuel* **29**, 524–539.

Carlsson G. E. (1974) Bite force and chewing efficiency. In: *Frontiers of Oral Physiology* (Edited by Kawamura Y.) Vol. 1, pp. 265–292. Karger, Basel.

Dahlberg B. (1942) The masticatory effect. *Acta med. scand.* **112**, suppl. 139, 1–156.

Edlund J. and Lamm C. J. (1980) Masticatory efficiency. *J. oral Rehab.* **7**, 123–130.

Epstein B. (1947) The mathematical description of certain breakage mechanisms leading to the logarithmico-normal distribution. *J. Franklin Inst.* **244**, 471–477.

Gaudin A. M. and Meloy T. P. (1962) Model and a comminution distribution equation for single fracture. *AIME Trans.* **223**, 40–43.

Glas H. W. van der, Bilt A. van der, Olthoff L. W. and Bosman F. (1985) Selection and breakage processes in the comminution of food *J. oral Rehab.* **12**, 551–552.

Lucas P. W. and Luke D. A. (1983a) Methods for analysing the breakdown of food in human mastication. *Archs oral Biol.* **28**, 813–819.

Lucas P. W. and Luke D. A. (1983b) Computer simulation of the breakdown of carrot particles during human mastication. *Archs oral Biol.* **28**, 821–826.

Manley R. S. and Braley L. C. (1950) Masticatory performance and efficiency. *J. dent. Res.* **29**, 448–462.

Olthoff L. W., Bilt A. van der, Bosman F. and Kleizen H. H. (1984) Distribution of particle sizes in food comminuted by human mastication. *Archs oral Biol.* **29**, 899–903.

Olthoff L. W., Bilt A. van der, Boer A. de and Bosman F. (1986) Comparison of force-deformation characteristics of artificial and several natural foods for chewing experiments. *J. Texture Stud.* **17**, 275–289.

Rosin P. and Rammler E. (1933) Gesetzmassigkeiten in der Kornzusammensetzung des Zementes. *Zement* **31**, 427–433.